Chapter 17 – Logarithms to Base 10 (L Scale)

Recall the Laws of Logarithms:

(i) $\log ab = \log a + \log b$

(ii) $\log \frac{a}{b} = \log a - \log b$

(iii) $\log a^N = N \log a$

And the equivalent Logarithmic and Exponential form:

$\log_b N = L$ and $N = b^L$

17.1 Logarithms and Antilogarithms Using L and D scales.

(i.e. usual L scale on body of the slide rule)

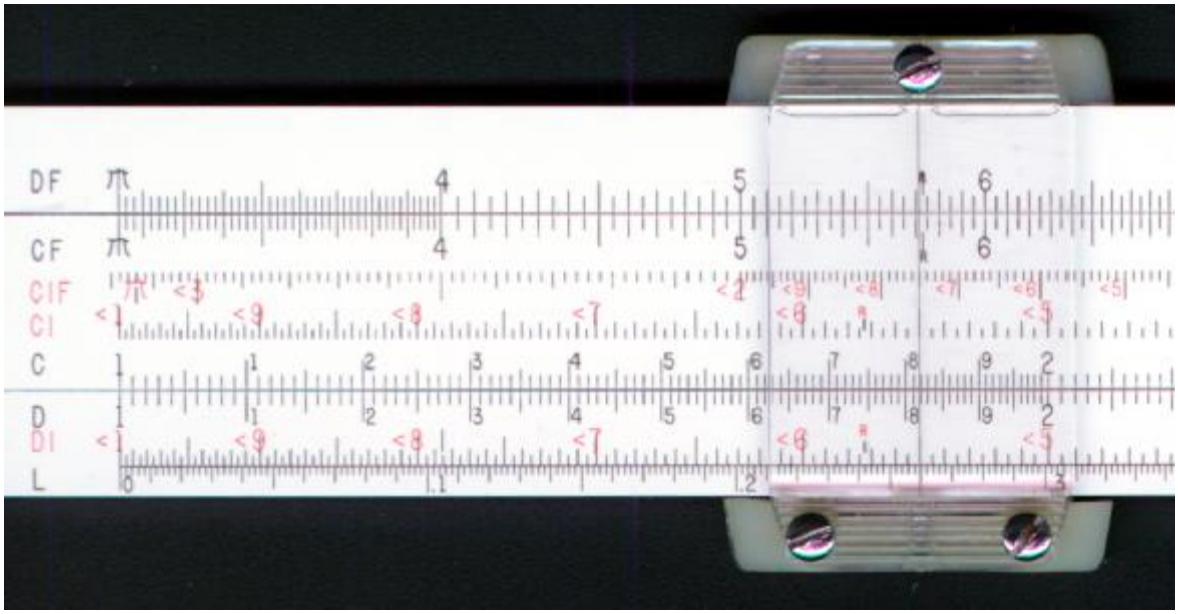

Fig 17-1

Example 1: $\log_{10} 1.82 = 0.26$ (Fig. 17-1)

(Or this could be stated $1.82 = 10^{0.26}$)

1. Set the hair line over 1.82 on the D scale.
2. Under the hair line read off 0.26 on the L scale as the answer.

Note:

(a) For numbers between 1 and 10 on the D scale, their logarithms are read directly off the L scale as the values between 0 and 1.

(b) For the logarithms of numbers outside the range 1 to 10, we have to decide their characteristic ourselves.

e.g. $\log_{10} 182 = 2.26$ or $\log_{10} 0.182 = -1.26$

The Slide Rule gives us only the mantissa, as do logarithm tables.

(c) If we have the logarithm of a number and wish to find the number, we work the opposite way, (i.e. from the L scale to the D scale.)

Example 2: antilogarithm of 3.26 = 1,820

(or this could be stated $10^{3.26} = 1{,}820$)

1. Set the hair line over 0.26 (i.e. mantissa only) on the L scale.
2. Under the hair line read off '182' on the D scale as the answer.

   $\therefore$ *answer* $= 1{,}820$ (as the characteristic is 3)

   (See exercise 17(a) at the end of 17.2 for problems)

Note:
As sines, cosines and tangents are found on the D scale, the value of log sin, log cos and log tan can be obtained by reading from the angle on the appropriate trigonometrical scale directly onto the L scale.

17.2 Logarithms and Antilogarithms Using L and W (Root) scales.
This is the system applicable to the Faber-Castell Slide Rules 2/83N, 62/83 etc. The L scale is on the slide and it is used in conjunction with the W scales. It is best to use the W'_1 and W'_2 scales instead of the W_1 and W_2 scales to avoid any error, should the slide be slightly displaced.

(i) For a number on the W'_1 scale, that is a number less than about 3.2 (3.5 for the 2/83 N), we read its logarithm off the L scale according to the numbers to the left of the graduations.

Example 1: $\log_{10} 1.385 = 0.1415$

1. Set the hair line over 1.385 on the W' scale.
2. Under the hair line read off 0.143 on the L scale (according to numbers to left of graduations) as the answer.

(ii) For a number on the W'_2 scale, that is a number greater than about 3 (2.8 for the 2/83N), we read its logarithm off the L scale according to the numbers to the right of the graduations.

Example 2: $\log_{10} 82.4 = 1.916$

1. Set the hair line over 82.4 the W'_2 scale.
2. Under the hair line read off 0.916 on the L scale (according to numbers to right of graduations) as the mantissa of the answer.

   $\therefore$ answer = 1.916 (as 82.4 is between 10 and 100).

Exercise 17(a)

(i) $\log_{10} 2.3$ =
(ii) $\log_{10} 46$ =
(iii) $\log_{10} 192$ =
(iv) $\log_{10} 0.67$ =
(v) $10^{0.477}$ =
(vi) $10^{1.699}$ =
(vii) $10^{2.45}$ =
(viii) $10^{5.855}$

Find X in the Following:

(ix) $\log_{10} X = 0.908$
(x) $\log_{10} X = -2.805$
(xi) $\log_{10} X = 2.015$
(xii) $\log_{10} X = 4.262$
(xiii) $\log_{10} X = 8.5$
(xiv) $\log_{10} X = 32$
(xv) $\log_{10} X = 0.065$
(xvi) $\log_{10} X = 2{,}500$

17.3 Raising Numbers to Powers and Solving Exponential Equations:
A. Raising a Number to Power. (A better method using LL scales is given in unit 19.)

Example 1: $33.4^{4.95} = 35{,}450{,}000$

Express as $\log_{10} 33.4^{4.95} = 4.95 \log_{10} 33.4$ (by Law III)

$= 4.95 \times 1.524$ (Using Slide Rule to Find logarithm)

$= 7.55$ (Multiply using Slide Rule)

(Use the Slide Rule again to obtain the antilog of 0.55 and position the decimal point according to the characteristic, 7.)

$\therefore$ answer = 35,450,000

B. Solving an Exponential Equation.

Example 2: solve $3^x = 5$ for x

If two quantities are equal, then their logarithms will be equal.

i.e. $\log_{10} 3^x = \log_{10} 5$

$\therefore x \log_{10} 3 = \log_{10} 5$ (evaluate each using Slide Rule)

$$\therefore x = \frac{\log_{10} 5}{\log_{10} 3} \text{ (divide using Slide Rule)}$$

$$= \frac{0.699}{0.477} \text{ (evaluate each using Slide Rule)}$$

$$\therefore x = 1.466$$

Exercise 17(b)

(i) $1.5^{7.8}$ =

(ii) $16.5^{2.5}$ =

(iii) $157^{0.68}$ =

(iv) $0.98^{3.6}$ =

Find x in the following:

(v) $2.3^x = 7.6$

(vi) $56^x = 29.5$

(vii) $0.8^x = 0.2$

(viii) $x^{1.5} = 12.5$

(Hint, write as $x = 12.5^{\frac{1}{1.5}}$)